

Brief information about the project

Name of the project	AP19676624 «Navier-Stokes-Voigt models governing the motion of homogeneous and nonhomogeneous non-Newtonian fluids»
Relevance	One of the most rapidly developing areas of mathematics is the mathematical analysis of various problems for nonlinear equations of non-Newtonian hydrodynamics. This project is devoted to the analytical and numerical study of the existence, uniqueness and qualitative properties of solutions to various direct and inverse problems for the nonlinear Navier-Stokes-Voigt equations and other related fluid mechanics equations with more complex rheological properties, describing the flow of homogeneous and nonhomogeneous incompressible fluids. Therefore, the study of inverse and direct problems for linear and nonlinear equations of Newtonian and non-Newtonian fluids, taking into account all possible properties, is an important and relevant area of research from both fundamental and practical points of view.
Purpose	The purpose of project is the investigation of new nonlinear inverse and boundary value problems for Navier–Stokes–Voigt models that describe incompressible homogeneous/nonhomogeneous flows with relaxation and elastic properties; the development of analytical and effective numerical methods for solving them and establishing the qualitative properties of their solutions.
Objectives	<ul style="list-style-type: none">➤ To establish the global existence and uniqueness of weak and strong generalized solution to the inverse source problem for the linearized integro-differential Navier-Stokes-Voigt system describing the flow of a viscous non-Newtonian incompressible fluid. The inverse problem consists of recovering a non-stationary source from the system using an additional overdetermination condition.➤ To prove the global-in-time existence and uniqueness of weak and strong solutions to the inverse source problem for the nonlinear integro-differential Navier-Stokes-Voigt equations with a special right-hand side.➤ To establish the local existence and uniqueness of weak and strong solutions to the nonlinear inverse source problem for the integro-differential Navier-Stokes-Voigt system.➤ To obtain asymptotic properties of the solution of the nonlinear inverse source problem for the Navier-Stokes-Voigt integro-differential system.➤ To investigate the well-posedness of the inverse problem with a final overdetermination condition the task of which is to find a source that depends only on a spatial variable for pseudoparabolic integro-differential equations.

	<ul style="list-style-type: none"> ➤ To implement an efficient algorithm for the numerical solution of an inverse source problem with final overdetermination condition for pseudoparabolic integral-differential equations. ➤ To prove the unique solvability of the inverse problem of recovering a time-dependent coefficient of the right-hand side for a pseudoparabolic integro-differential equation. ➤ To study numerical solutions of the inverse problem of retrieving a time-dependent coefficient of right-hand side for a pseudoparabolic integro-differential equation, which models Voigt fluid flows. ➤ To investigate the issues of existence and uniqueness of strong solutions of the direct initial-boundary value problem for the nonlinear integro-differential Navier-Stokes-Voigt system and establish the qualitative properties of the solutions. ➤ To study the unique solvability of the initial-boundary value problem for the Navier-Stokes-Voigt equations, which describe nonhomogeneous flows of viscous and incompressible fluids with elastic properties. In this problem, the initial density may eventually vanish over time in some subdomain, i.e. we work on the possibility that, at the initial moment, there might be a vacuum in some part of the spatial domain. ➤ To prove the existence, uniqueness and stability of generalized strong solutions of the inverse problem for the linearized Navier-Stokes-Voigt system. The inverse problem consists of finding the intensity of external forces depending on the spatial variable. ➤ To establish the existence and regularity of generalized solutions of a nonlinear initial-boundary value problem for integro-differential Navier-Stokes-Voigt equations with memory that describe nonhomogeneous flows of viscous and incompressible non-Newtonian fluids with elastic properties.
Expected and achieved results	<p>In 2023, the following new results were obtained:</p> <ul style="list-style-type: none"> ➤ The global existence and uniqueness of weak and strong generalized solution to the inverse source problem for the linearized integro-differential Navier-Stokes-Voigt system, which describes the flow of a viscous non-Newtonian incompressible fluid, has been established. This inverse problem consists of restoring a non-stationary source from the system using an additional integral overdetermination condition. ➤ The global-in-time existence and uniqueness of weak and strong solutions to the inverse source problem for a nonlinear integro-differential Navier-Stokes-Voigt equations with a special right-hand side are proved. ➤ The local existence and uniqueness of weak and strong solutions of nonlinear inverse source problem for

	<p>the integro-differential Navier-Stokes-Voigt system are established.</p> <ul style="list-style-type: none"> ➤ Asymptotic properties of solution of the nonlinear inverse problem for the integro-differential Navier-Stokes-Voigt system are obtained. ➤ The well-posedness of the inverse problem of finding a source that depends only on a spatial variable for pseudoparabolic integro-differential equations with a final overdetermination condition is investigated. ➤ An effective algorithm for the numerical solution of the inverse source problem with the final overdetermination condition for integro-differential pseudoparabolic equation has been created. <p>The following results are expected in 2024-2025:</p> <ul style="list-style-type: none"> ➤ The unique solvability of the inverse problem of recovering a time-dependent coefficient of the right-hand side for an integro-differential pseudoparabolic equation will be proved. ➤ Numerical solutions of the inverse problem of recovering the time-dependent coefficient of the right-hand side of the pseudo[arabolic integro-differential equation that models Voigt fluid flows will be investigated. ➤ The questions of the existence and uniqueness of strong solutions of the direct initial-boundary value problem for the nonlinear integro-differential Navier-Stokes-Voigt system will be investigated and the qualitative properties of the solutions will be established. ➤ The unique solvability of the initial-boundary value problem for the Navier-Stokes-Voigt equations, which describe nonhomogeneous flows of viscous and incompressible fluids with elastic properties, will be established. In this problem, the initial density may disappear over time in a certain subdomain, i.e. we are working on the possibility that at the initial moment a vacuum may exist in some part of the spatial domain. ➤ The existence, uniqueness and stability of strong generalized solutions of the inverse problem for the linearized Navier-Stokes-Voigt will be proved. The inverse problem consists of finding the intensity of external forces depending on the spatial variable. ➤ The existence and regularity of generalized solutions of a nonlinear initial-boundary value problem for integro-differential Navier-Stokes-Voigt equations with memory, describing nonhomogeneous flows of viscous and incompressible non-Newtonian fluids with elastic properties, will be established.
<p>Research team members with their identifiers (Scopus Author ID, Researcher ID, ORCID, if</p>	<p>1. Khompysh Khonatbek, candidate of physics and mathematics, associate professor. h-index: Scopus –9, Web of Science –8. <u>ORCID: 0000-0002-5525-111X</u>, Scopus author ID: 55785395700.</p>

<p>available) and links to relevant profiles</p>	<p>2. Hermenegildo Borges de Oliveira, PhD, professor. h-index: Scopus –10, Web of Science –9. <u>ORCID: 0000-0001-9053-8442</u>, Scopus author ID: 7004475473.</p> <p>3. Abylkairov Undasyn Utegenovich, candidate of physics and mathematics, associate professor. h-index: Scopus–3; Web of Science –1; ORCID: <u>0000-0002-1191-5519</u>, Scopus Author ID: 56841525200.</p> <p>4. Kabidoldanova Assem Altaykyzy, candidate of physics and mathematics. h-index: Scopus –1, Web of Science –1. ORCID: <u>0000-0001-6375-9805</u>, Scopus Author ID: 55321771300.</p> <p>5. Kenzhebai Khanat, PhD student. h-index: Scopus –1, Web of Science –1. ORCID: <u>0000-0001-6787-128X</u>, Scopus author ID: 57381328000.</p> <p>6. Nugymanova Nursuale Kuanyshbekovna, PhD student. h-index: Scopus –1, Web of Science –1. Scopus author ID: 57987744400.</p> <p>7. Shakir Aidos Ganizhanuly, PhD. h-index: Scopus – 1, Web of Science –1. ORCID: <u>0000-0001-8572-0776</u>, Scopus Author ID: 57887170500.</p> <p>8. Mukhambetkaliyev Murat Bakytzhanovich, PhD student.</p> <p>9. Shazyndayeva Moldir Kassymkyzy, PhD student.</p>
<p>List of publications with links to them</p>	<p>2023 year: Web of Science and Scopus</p> <p>1. Khompysh Kh., Shakir A., Kabidoldanova A. Inverse problems for nonlinear Navier-Stokes-Voigt system with memory//Chaos, solitons and fractals. —177(12). —2023. Web of Science: Q1, Scopus: 99%.</p> <p>2. Khompysh Kh., Shakir A.G. Inverse problems for Kelvin-Voigt system with memory: global existence and uniqueness. Lobachevskii journal of mathematics. — 44(10). — p. 4348–4359. — 2023. Web of Science: Q3, Scopus: 56 %.</p> <p>3. Khompysh Kh., Nugmanova N.K. Kelvin-Voigt equations with memory: existence, uniqueness and regularity of solutions//Bulletin of the Karaganda university. Mathematics series. —112 (4). — 2023. Web of Science: Q3, Scopus: 35%.</p> <p>Materials of conference</p> <p>1. de Oliveira H.B., Khompysh Kh., Shakir A.G. Navier-Stokes-Voigt equations governing density dependent flows with vacuum//Materials of the conference: Functional Analysis in Interdisciplinary Application, 2-7 October. – Antalya: 2023.–P.89.</p> <p>2. de Oliveira H.B., Khompysh Kh., Shakir A.G. Navier-Stokes-Voigt system governing density dependent flows with vacuum//Materials of the conference: Non-local boundary value problems and related problems of mathematical biology, informatics and physics, 4-8 December. – Nalchik: 2023. –P.329.</p>

	<p>3. Khompysh Kh. Nonlocal problems for pseudo-parabolic equation with Laplace operator//Materials of the conference: Non-local boundary value problems and related problems of mathematical biology, informatics and physics, 4-8 December. – Nalchik: 2023. –P.342.</p> <p>4. Khompysh Kh., Shazyndayeva M.K. An inverse problem for integro-differential pseudoparabolic equation//Materials of the conference: Non-local boundary value problems and related problems of mathematical biology, informatics and physics, 4-8 December. – Nalchik: 2023. –P.343.</p> <p>5. Mukhambetkaliyev M.B. Time dependent inverse source problem for pseudoparabolic equation with memory//Materials of the conference: Non-local boundary value problems and related problems of mathematical biology, informatics and physics, 4-8 December. – Nalchik: 2023. –P.357.</p>
Patents	-